2026 Sep. 16 シンポジウム「機械学習と計算アルゴリズム - 計算物理と関連分野における進展」16pL1225-5

Quantum-inspired Generative Models

— Born Machines and Tensor Networks for Learning Probability Distributions —

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Tensor network = a network of tensor contractions

Ex. Quantum amplitudes of a quantum state

N qubits $$|\Psi\rangle = \sum_{i_1=0,1} \cdots \sum_{i_n=0,1} T_{i_1 \cdots i_n} |i_1 \cdots i_n\rangle$$

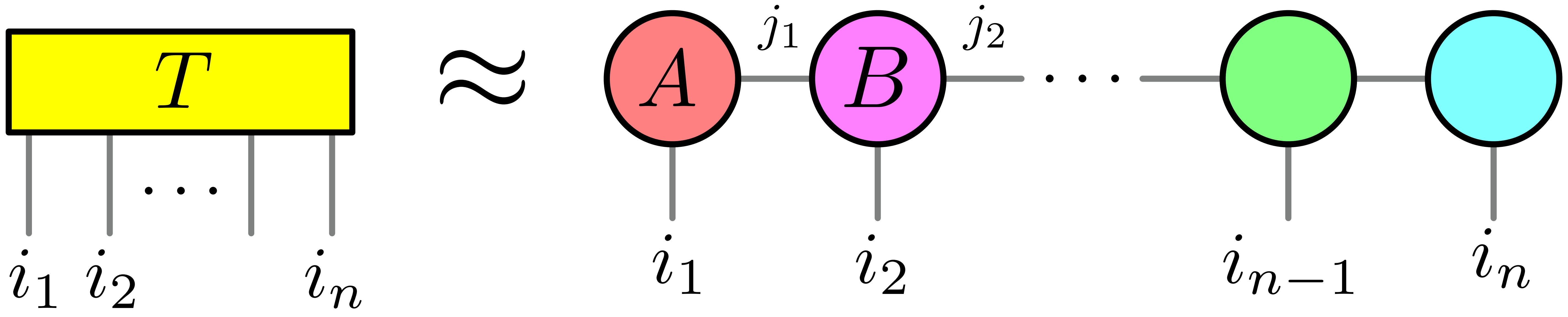
Tensor $T_{i_1 i_2 \cdots i_n} \Longleftrightarrow \#T = 2^n$ Exponentially large

Tensor decomposition

$$T_{i_1 i_2 \cdots i_n} = A_{i_1 j_1} B_{j_1 i_2 j_2} \cdots$$

If $D = \#j\,,\ \ \#T \sim O(n \times D^2)$

Exponential compression
when D remains small



Tensor network : MPS (matrix product state; also known as tensor train)

Applications to machine learning

Compact representation of
high-dimensional tensors

- ✦ Neural-network parameter compression
- ✦ Data compression

Quantum-inspired algorithms

- ✦ Classification (tensor-network kernel)
- ✦ **Generative model**

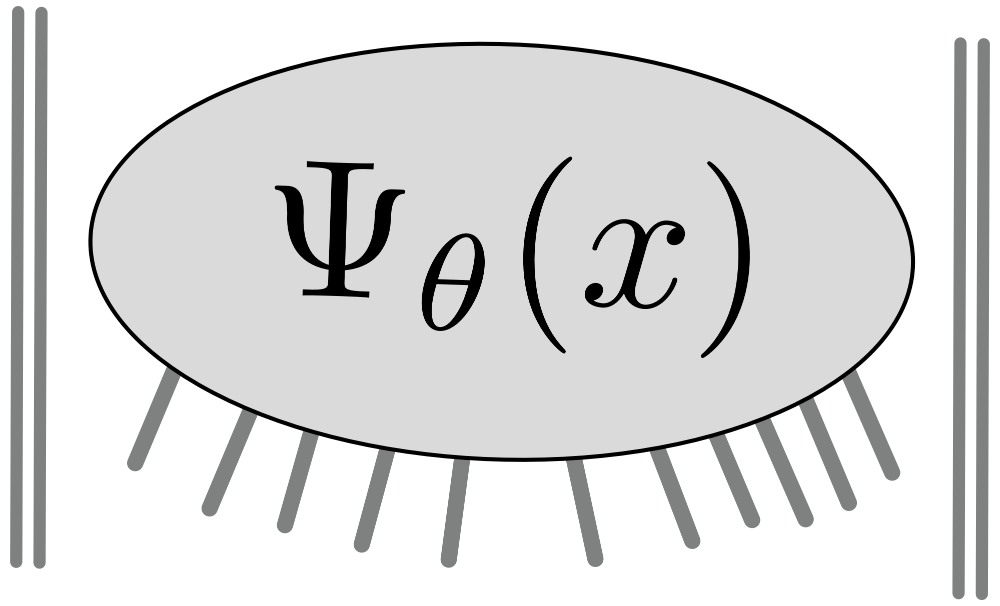
Born machine: a quantum-inspired generative model

Generative model

$$P_\theta(x) \sim$$

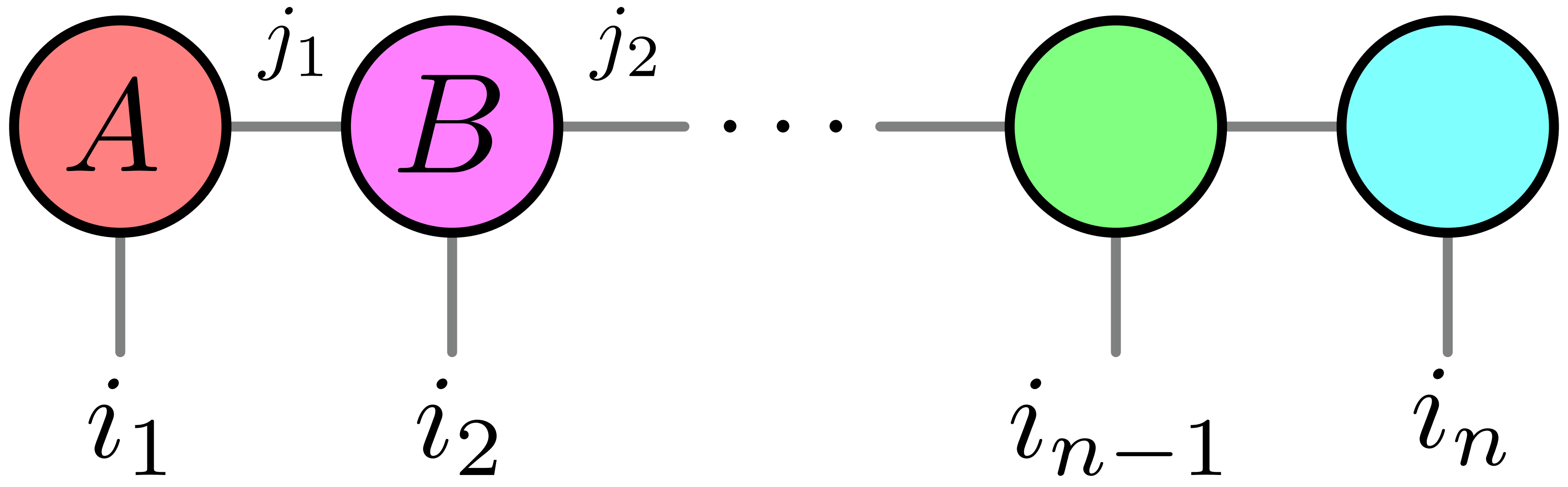
Data distribution

$$= \left\| \Psi_\theta(x) \right\|^2$$



θ: parameters

Wave function

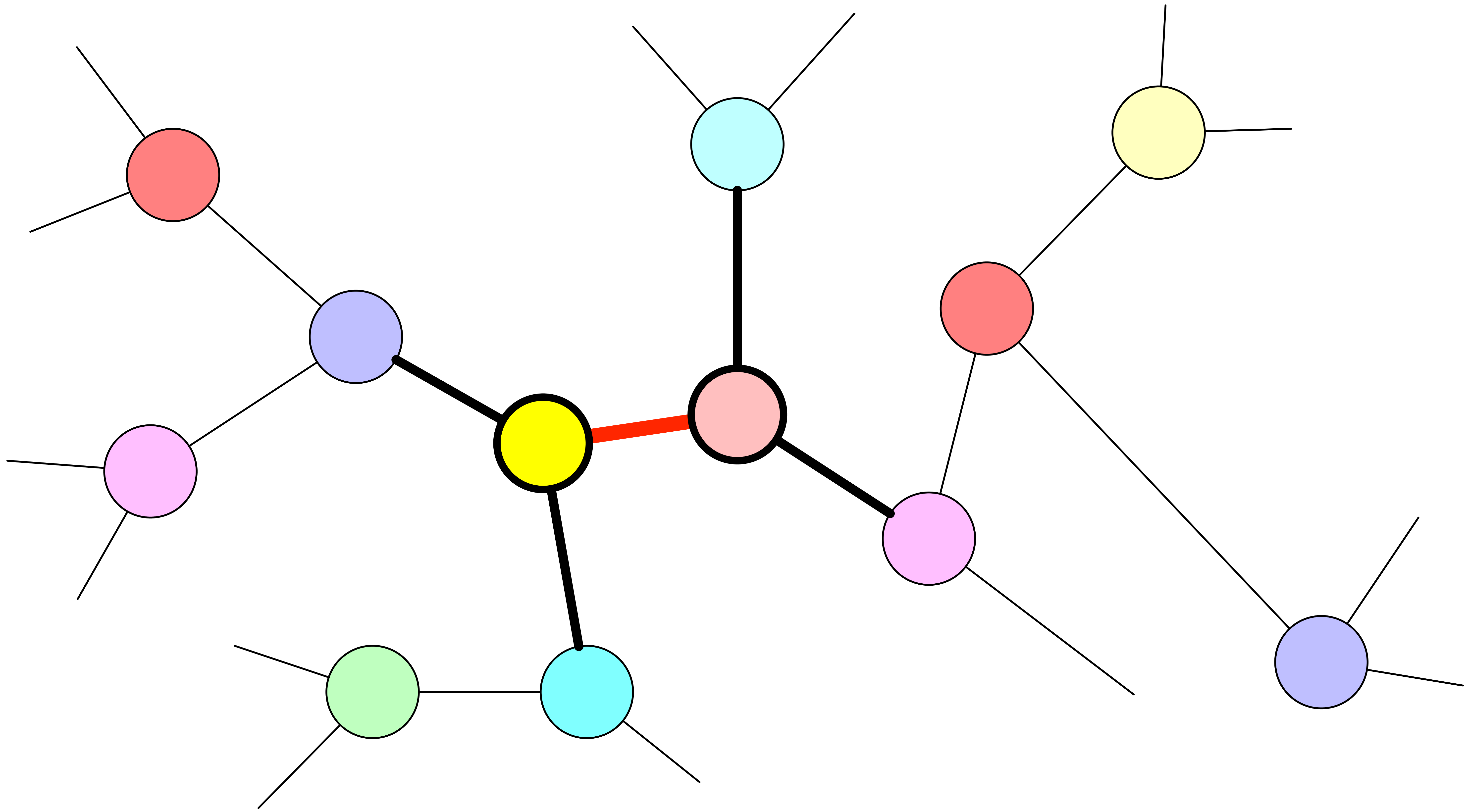


Han, *et al.* PRX 2018

Fixed network structure: MPS

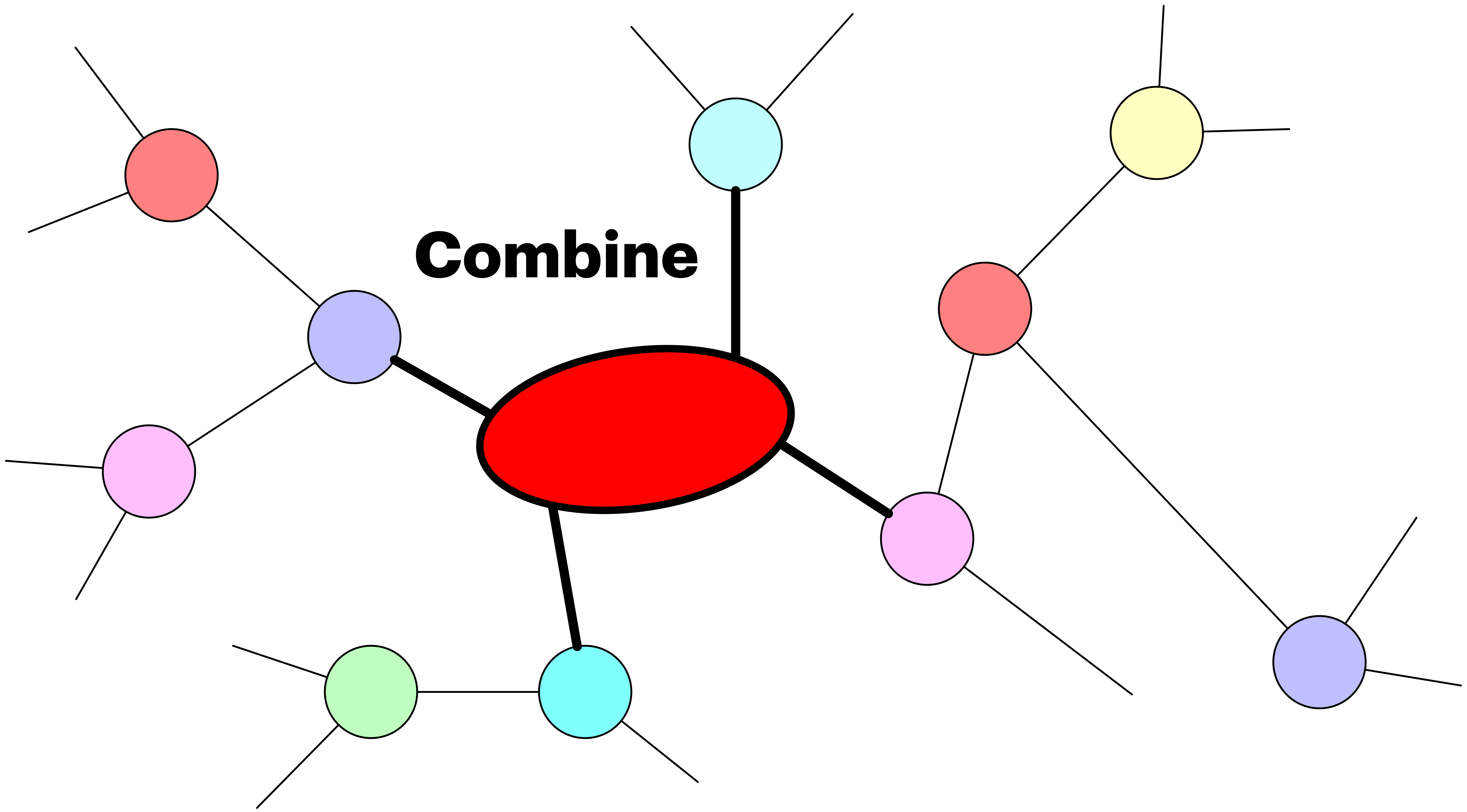
But performance depends strongly on the network structure.

Adaptive tensor tree algorithm (ATT)



Harada, Okubo, Kawashima, MLST 2025

Adaptive tensor tree algorithm (ATT)

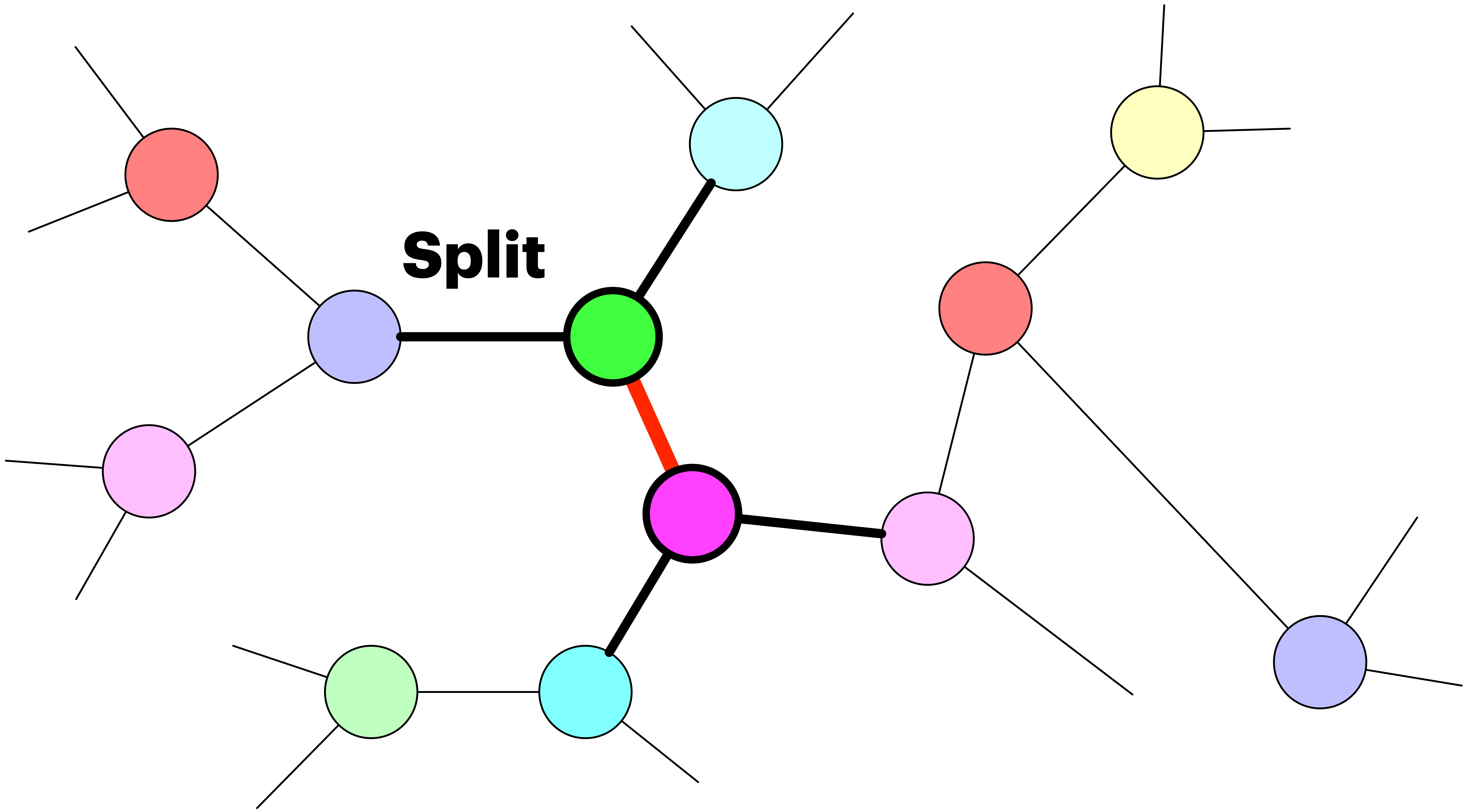


Harada, Okubo, Kawashima,
MLST 2025

Adaptive tensor tree algorithm (ATT)

Avoid information bottlenecks: $\mathrm{MI}(A:B) \leq \log D$

Choose the split with the smallest bond mutual information



Harada, Okubo, Kawashima, MLST 2025

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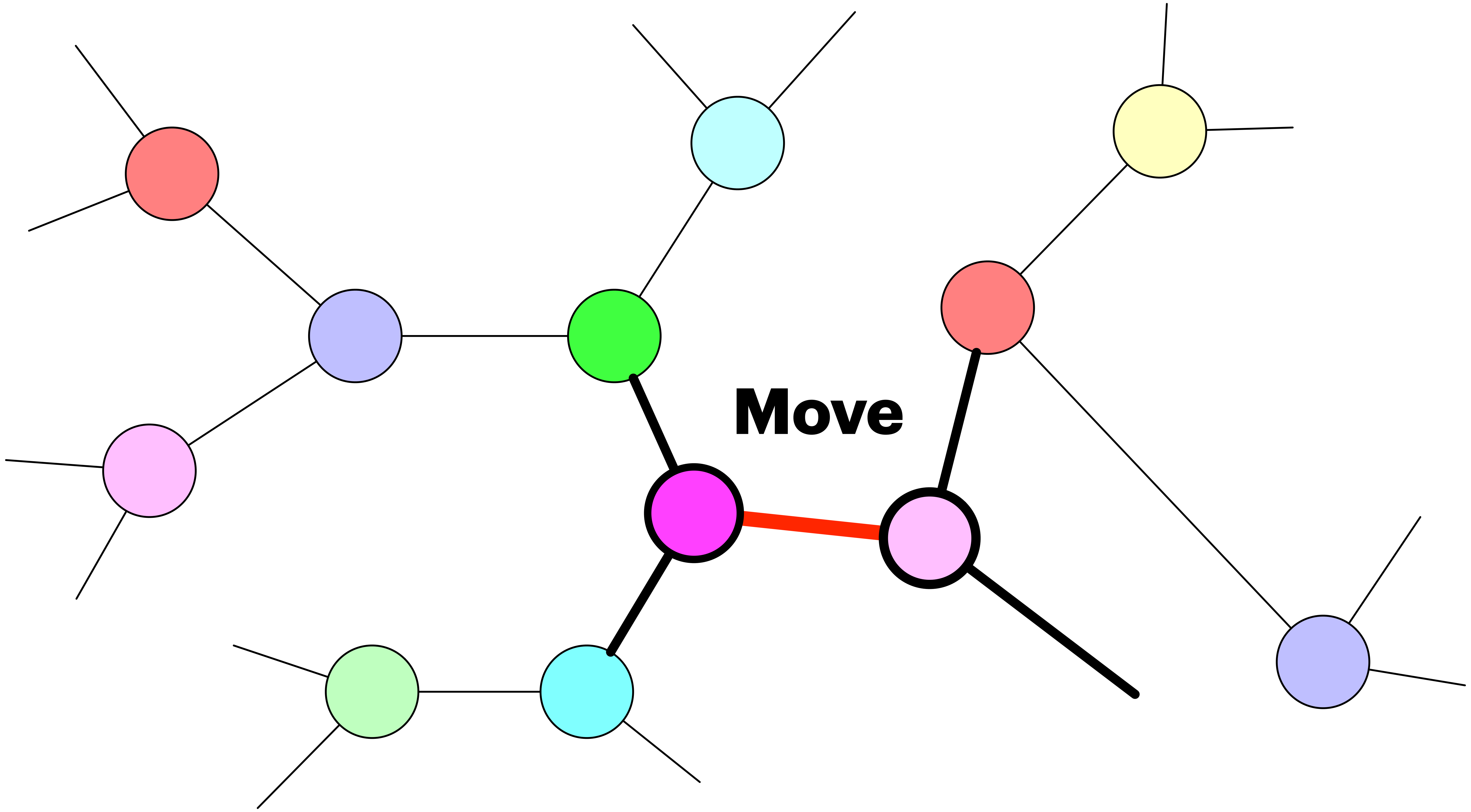


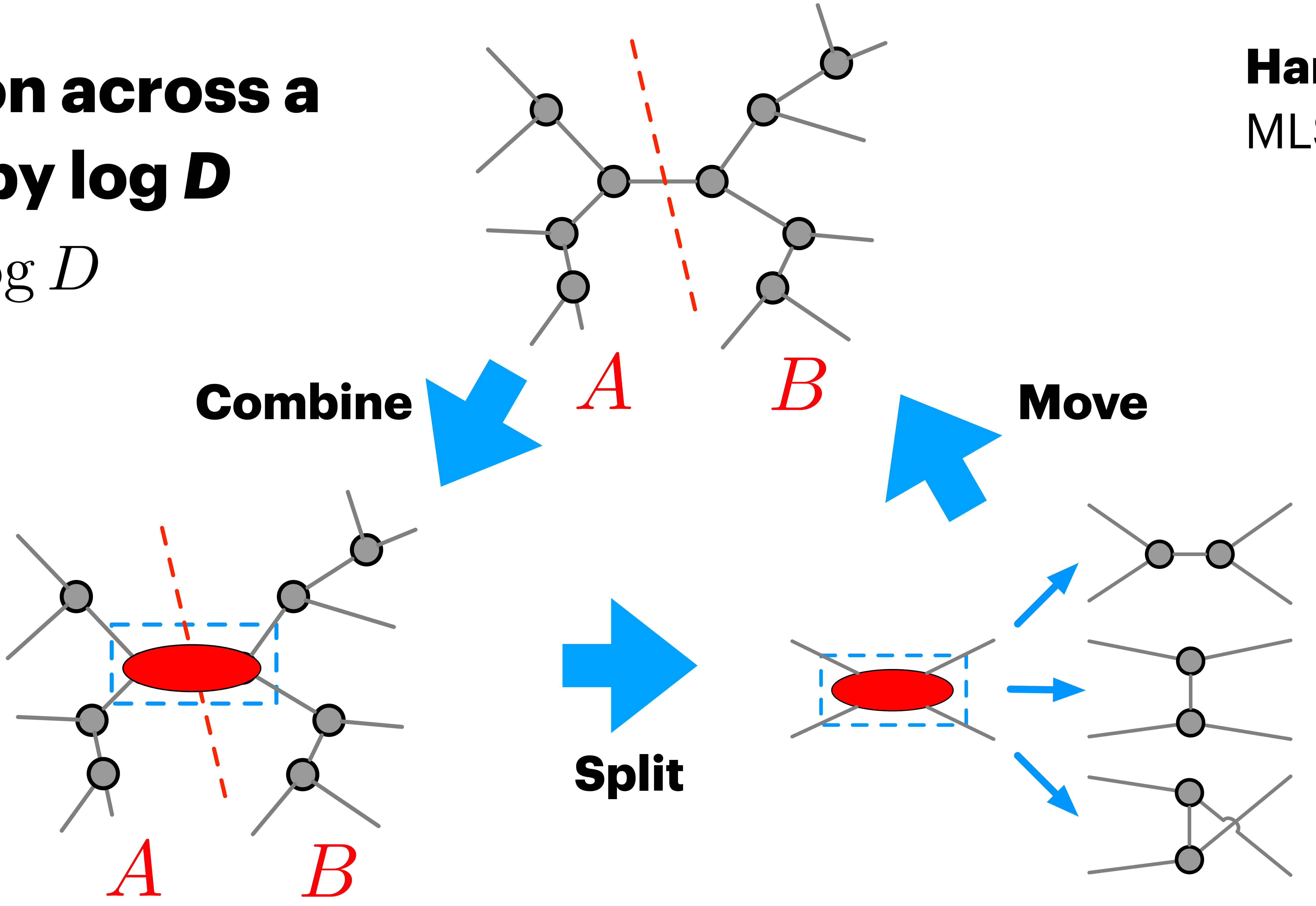
Harada, Okubo, Kawashima,
MLST 2025

Adaptive tensor tree algorithm (ATT)

Mutual information across a bond is bounded by log *D*

$$\mathrm{MI}(A:B) \leq \log D$$

Harada, Okubo, Kawashima, MLST 2025



Optimize the merged tensor by minimizing the negative log-likelihood (NLL)

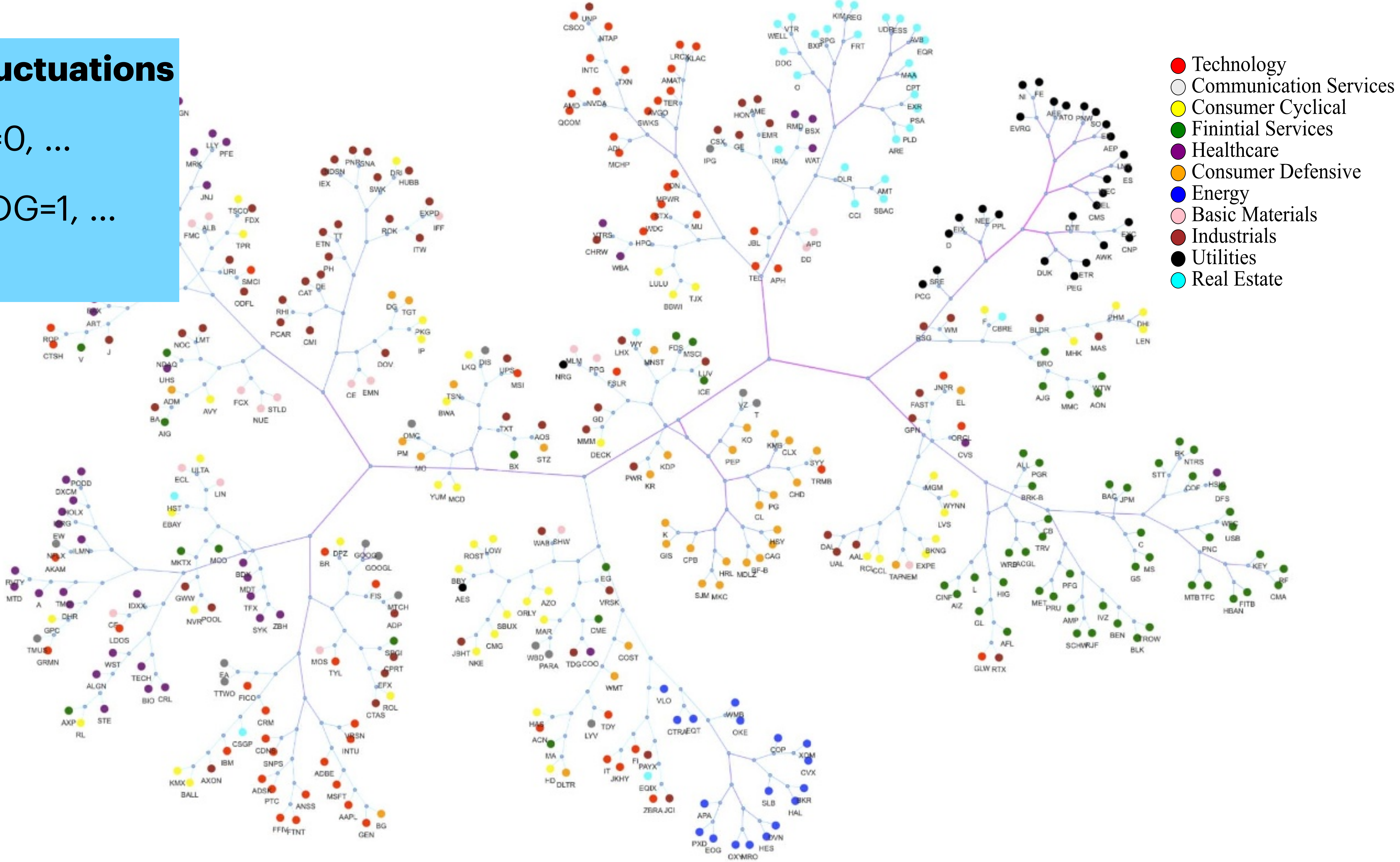
Update the connection by selecting the split with the smallest MI

Joint optimization of tensor parameters and network topology

Example: S&P 500

Samples : stock price fluctuations

- Day x: AAPL=1, GOOG=0, ...
- Day (x+1): AAPL=1, GOOG=1, ...
- Day (x+2): ...



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Samples : stock price fluctuations

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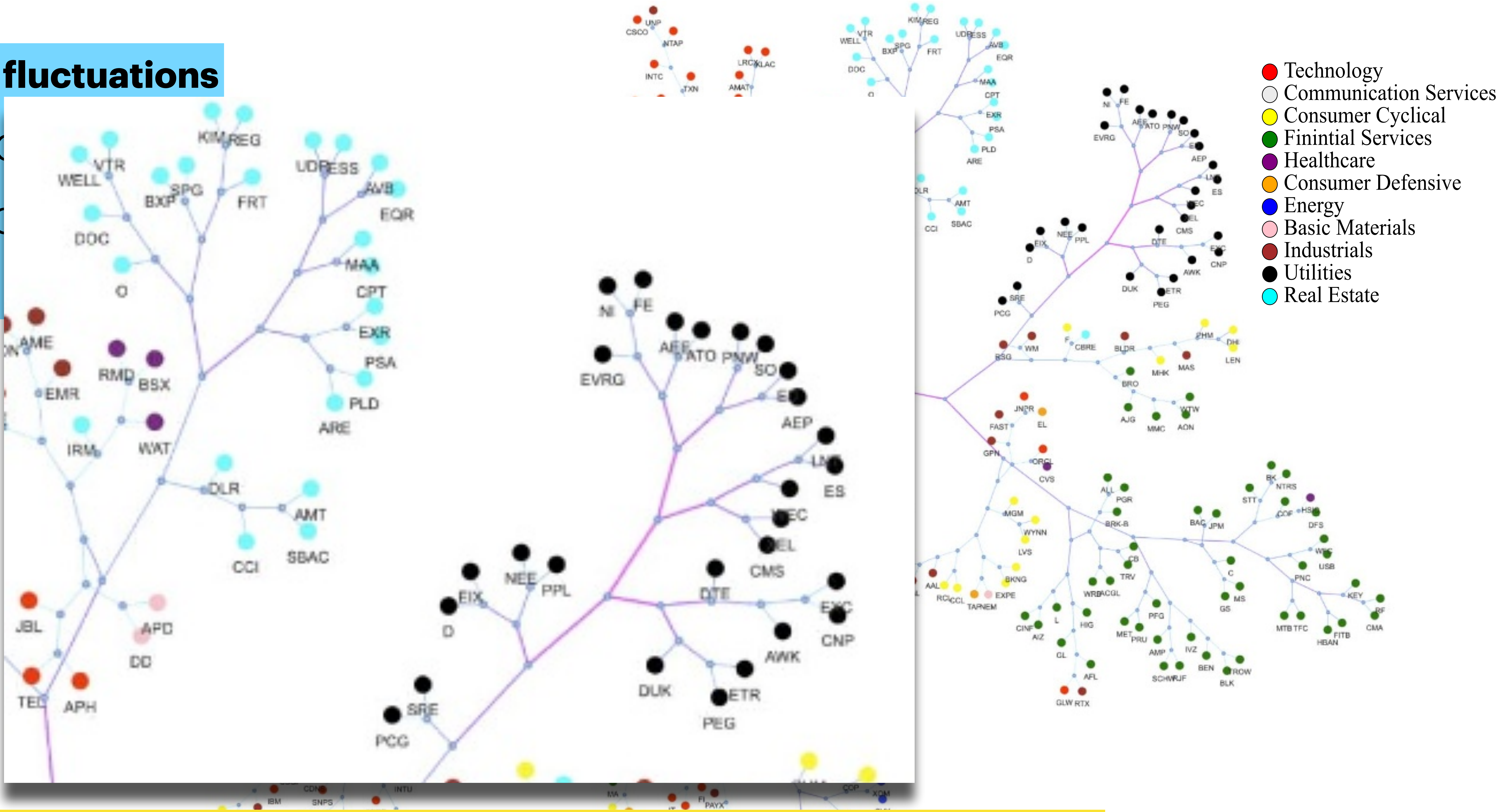


Sector-wise organization emerges in the learned network structure

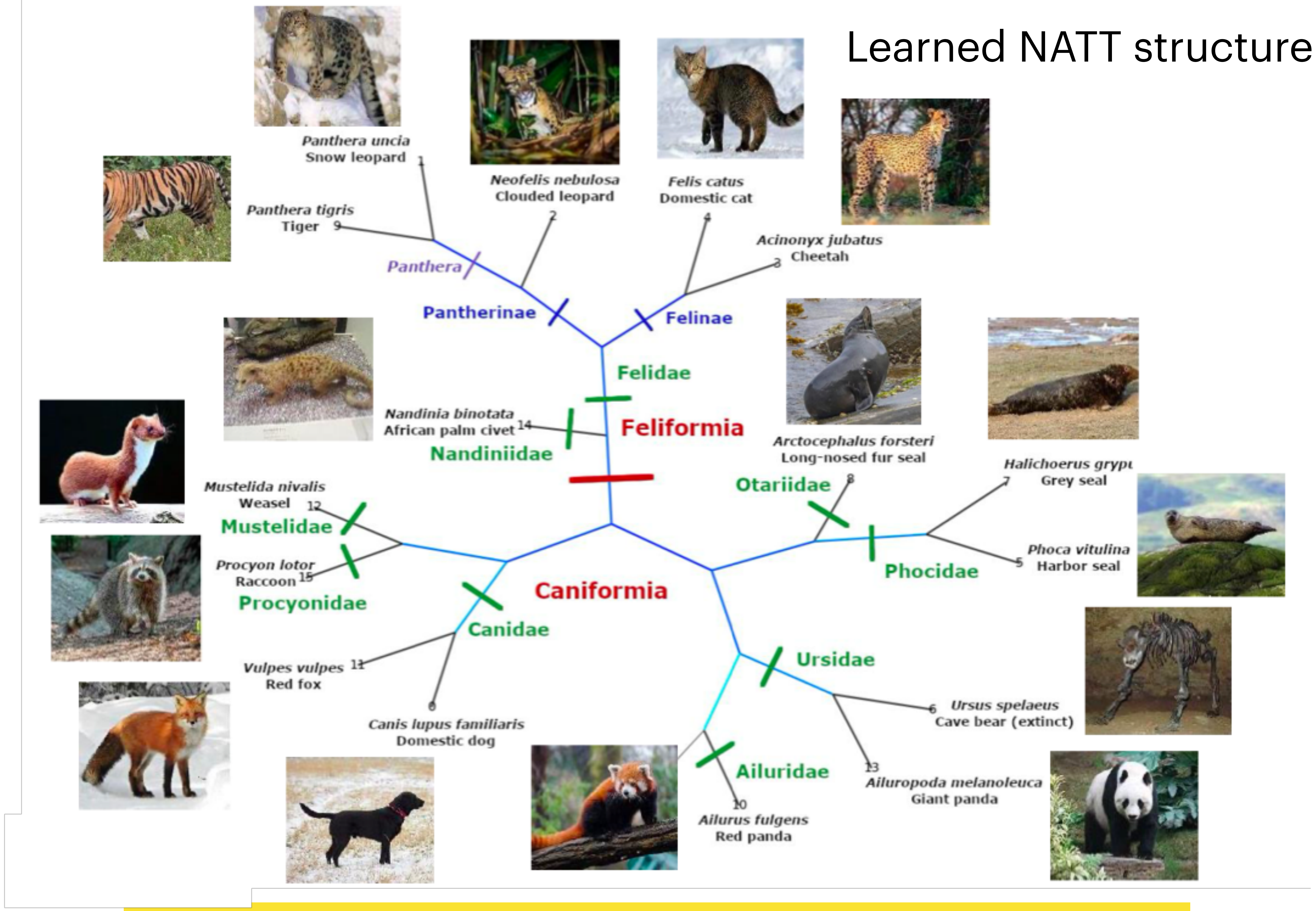
Non-negative Adaptive Tensor Tree Algorithm (NATT)

Non-negative tensor tree

- Single-layer tensor tree
- All tensor elements are non-negative
- Interpreted as a probabilistic graphical model
- SVD → non-negative matrix factorization (NMF)

Akamatsu, **Harada**, Okubo, and Kawashima, MLST 2026

Learned NATT structure



A phylogeny-like tree emerges from mitochondrial DNA sequences of 16 species

From Tensor-Network Optimization to Structure Learning

Adaptive tensor trees (ATT / NATT)
mutual information, topology, local reconnection

Graphical models
dependency structure, latent variables

Probabilistic circuits
tractable sum-product computation, structure learning
(Yang, et al. AISTATS 2023)

Common question: What network structure best represents the dependencies in data?

This suggests a connection to structure learning in graphical models and probabilistic circuits.

Summary: From Quantum Many-Body Physics to Machine Learning

Quantum many-body physics
MPS / tensor networks

→

Generative modeling
Born machine

↙

Adaptive structure
ATT

→

Probabilistic structure learning
NATT → graphical models; links to PCs

Network structure itself can be learned from data.

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